Differential Cohomology Theories are Sheaves on Manifolds

Idea Include some differential form data into integral cohomology to retain geometric information:

$\Omega^\bullet_M$ is a complex of sheaves and the cohomology of $(\Omega^\bullet(M), d)$ is $H^\bullet(M, \mathbb{R})$.

$$\begin{tikzcd}
\hat{H}^\bullet(M) \arrow[r, "I"] \arrow[d, "R"] & H^\bullet(M, \mathbb{Z}) \arrow[d] \\
\Omega^\bullet_{cl}(M) \arrow[r] & H^\bullet(M, \mathbb{R})
\end{tikzcd}$$

"homotopy pullback"

Goals for today and next week

1. Develop abstract framework of differential cohomology as "sheaves of spectra on manifolds"

2. Reproduce the above square in high generality:

Th generalized cohomology theory + "form data" + "a way to combine them"

$$\Updownarrow$$

differential cohomology theory

3. Rediscover the hexagon diagram

4. The homotopy formula and integration

As time permits:

a) Chern-Simons theory as a differential refinement of Chern-Weil theory

b) Differential K-theory

c) Twisted differential cohomology theories

$\boxed{\text{Today}}$

1. The functor of points and sheaves
2. Prerequisites: ∞-categories & spectra
3. Differential cohomology theories
4. Homotopy invariant sheaves

1. The functor of points perspective (a.k.a. Yoneda)

$N \in \mathrm{Mfd} \Rightarrow F := C^\infty(-,N) : \mathrm{Mfd}^{op} \to \mathrm{Set}$

functor

$F(pt) = N^\delta$ underlying set

$F(M) = C^\infty(M,N)$ "probing N with M", "plots"

Def A <u>smooth set</u> is a sheaf $F : \mathrm{Mfd}^{op} \to \mathrm{Set}$

(i.e. $\forall M \in \mathrm{Mfd}$, $F|_M$ is a sheaf)

<u>Example</u> 1. A diffeological space is a "concrete" smooth set

2. $A \in \mathrm{Set}$. Then $X \mapsto A$ is not a sheaf.

Its sheafification $F := \mathrm{Const}\, A$ has

$F(X) = \{ X \to A \text{ locally constant} \}$

Idea $\mathcal{C}$ category, then a smooth object in $\mathcal{C}$ is a sheaf

$F \in Sh(Mfd; \mathcal{C})$

Very generally, a differential cohomology

theory F could be a 'smooth spectrum'

$F \in Sh(Mfd, Sp)$ "category" of spectra $\approx$ cohomology theories

Caveat have to treat spectra with proper homotopical viewpoint

lim $\rightsquigarrow$ holim

diagram commutes $\rightsquigarrow$ diagram commutes up to homotopy + coherence data

so if $f_1 : M_1 \to M_2$, $f_2 : M_2 \to M_3$ are smooth maps

$$F(f_2 \circ f_1) \overset{H}{\sim} F(f_1) \circ F(f_2)$$

data! + higher coherences ...

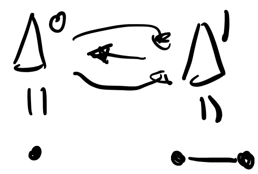
Capture all coherence: Sp is an ∞-category

& F an ∞-functor $Mfd^{op} \to Sp$

2. Recollections of ∞-categories

For us $(\infty,1)$-category $=$ ∞-category $=$ quasi-category

Def Δ simplex category:



objects: n-simplices $\{0,\dots,n\}$ $\forall n$

morphisms: $\leq$-preserving maps

A simplicial object in a category $\mathcal{C}$ is a functor

$$\Delta^{op} \to \mathcal{C}$$

A cosimplicial object in $\mathcal{C}$ is a functor

$$\Delta \to \mathcal{C}$$

Example If $\mathcal{C}$ is a category, then the nerve

$$N(\mathcal{C})_n := \{ c_0 \xrightarrow{f_1} c_1 \to \dots \xrightarrow{f_n} c_n \ \text{composable} \}$$

is a simplicial set.

It has the property that every "inner horn" has a unique filler:

$$\begin{tikzcd} & c_1 \arrow[dr, "f_2"] \arrow[d, phantom, "\Downarrow"] & \\ c_0 \arrow[ur, "f_1"] \arrow[rr, dotted] & {} & c_2 \end{tikzcd} \qquad \begin{tikzcd} & c_1 \arrow[dr, dotted, "\times"] & \\ c_0 \arrow[ur] \arrow[rr] & & c_2 \end{tikzcd}$$

This defines a Fully faithful functor $\mathrm{Cat} \to s\mathrm{Set}$

Def An <u>∞-category</u> (or <u>quasi-category</u> or <u>weak Kan complex</u>) is a simplicial set s.t. every inner horn has a (not necessarily unique) filler.

Δ^0 $\quad\quad\quad$ Δ^1

<u>Dictionary</u>

$$\begin{tikzcd} \mathcal{C}_0 \arrow[r, "\text{identity}" description] & \mathcal{C}_1 \arrow[l, bend right, "\text{source}"'] \arrow[l, bend left, "\text{target}"] \end{tikzcd}$$

objects $\quad\quad\quad\quad$ morphisms

filler of $\begin{tikzcd} & \bullet \arrow[dr] & \\ \bullet \arrow[ur] \arrow[rr, dotted] & & \bullet \end{tikzcd}$ = choice of composition (not unique!)

Prop $\exists$ left adjoint to the nerve

$$\begin{tikzcd} s\mathrm{Set} \arrow[r, bend left, "h"] & \mathrm{Cat} \arrow[l, bend left, "N"] \end{tikzcd}$$

If $\mathcal{C}$ is an ∞-category, we call $h\mathcal{C}$ the <u>homotopy category</u>

There is a generalization of the nerve for categories enriched in simplicial sets Cat_Δ

$N : Cat_\Delta \to sSet$

called the homotopy-coherent nerve.

For Kan-enriched, get an ∞-cat.

Topological categories are related to Cat_Δ by geometric realization & singular complex functors

Example Spaces (CW-complexes) has internal homs, so the homotopy-coherent nerve makes it into an ∞-category.

Def/Prop $\mathcal{C}, \mathcal{D}$ ∞-categories.

Then $Fun(\mathcal{C}, \mathcal{D}) := Map_{sSet}(\mathcal{C}, \mathcal{D}) \in sSet$

is the ∞-category of functors

Note The category of ∞-categories is enriched over simplicial sets

There is an ∞-category of (small) ∞-categories

Def For $x, y \in \mathcal{C}_0$, the simplicial set $\mathrm{Hom}_{\mathcal{C}}(x,y)$ of <u>morphisms</u> is the pull back

$$\begin{tikzcd} \mathrm{Hom}_{\mathcal{C}}(x,y) \arrow[r] \arrow[d] & \mathrm{Fun}(\Delta^1, \mathcal{C}) \arrow[d, "(ev_0, ev_1)"] \\ * \arrow[r, "(x,y)"] & \mathcal{C} \times \mathcal{C} \end{tikzcd}$$

<u>Next</u> one can define (co)limits, adjoint functors, ...

<u>Example</u> $y \in \mathcal{C}_0$ is final if $\mathrm{Hom}_{\mathcal{C}}(x,y)$ is contractible $\forall x \in \mathcal{C}_0$.

<u>Def</u> $\mathcal{C}$ is <u>stable</u> if

1. it admits finite limits & colimits
2. its final object is an initial object (zero object)
3. A diagram $(\Delta^1 \times \Delta^1 \to \mathcal{C})$

$$\begin{tikzcd} x \arrow[r] \arrow[d] & y \arrow[d] \\ z \arrow[r] & w \end{tikzcd}$$

is a pull back iff it is a pushout

Define <u>suspension</u> and <u>loop space</u> by

$$\begin{tikzcd} X \arrow[r] \arrow[d] & 0 \arrow[d] \\ 0 \arrow[r] & \Sigma X \end{tikzcd} \qquad \begin{tikzcd} \Omega X \arrow[r] \arrow[d] & 0 \arrow[d] \\ 0 \arrow[r] & X \end{tikzcd}$$

Example $Sp = \lim\left(\cdots \xrightarrow{\Omega} \mathrm{Spaces} \xrightarrow{\Omega} \mathrm{Spaces} \right)$

is stable.

$X \in \mathrm{Space}$

$\sim$ Infinite loop spaces $\Omega E_n \xrightarrow{\sim} E_{n-1}$

$\Rightarrow H^n(X, E) := \pi_0 \mathrm{Map}(\Sigma^\infty X, \Sigma^n E)$

is a generalized cohomology theory

Th Every generalized cohomology theory comes from a spectrum

Def If $f: X \to Y$ is a map of spectra, the fiber and cofiber are

$$\begin{tikzcd} \mathrm{fib}\, f \arrow[r] \arrow[d] & X \arrow[d, "f"] \\ 0 \arrow[r] & Y \end{tikzcd} \qquad \begin{tikzcd} X \arrow[r, "f"] \arrow[d] & Y \arrow[d] \\ 0 \arrow[r] & \mathrm{cofib}\, f \end{tikzcd}$$

Recall The Dold-Kan correspondence gives an equivalence

$DK: Ch_{\geq 0} \xrightarrow{\sim} sAb$

simplicial abelian groups

For unbounded chain complexes we can do

$DK_n : Ch \xrightarrow{\tau_{\geq n}} Ch_{\geq n} \overset{\text{shift}}{\simeq} Ch_{\geq 0} \xrightarrow[\sim]{DK} sAb \xrightarrow{\text{forget}} sSet$

Up to weak equivalences, $DK_n(A^\bullet) \in sSet$ assemble into a spectrum & we get a functor of ∞-categories

$$DK : Ch \to Sp$$

with the property that the chain complex

$$\dots \to 0 \to A[0] \to 0 \to \dots$$

is mapped to the Eilenberg-Maclane spectrum HA

3. Differential Cohomology Theories

We can now define differential cohomology theories as functors

$$F : \mathrm{Mfd}^{op} \to \mathrm{Sp}$$

that satisfy a kind of sheaf condition.

Recall $F : \mathrm{Mfd}^{op} \to \mathrm{Set}$ sheaf condition:

$$F(M) \xrightarrow{\sim} \lim\Big(\prod_{i_1, i_2} F(U_{i_1} \cap U_{i_2}) \leftleftarrows \prod_i F(U_i) \Big)$$

for $\{U_i\}$ good open cover.

How do we generalize this to a higher categorical target?

Example For G discrete group,

we want the sheafification of the constant presheaf with value $BG \in \mathrm{Spaces}$ to be

$$F(M) = \mathrm{Maps}(M, BG)$$

$$= \{\text{principal } G\text{-bundles on } M\}$$

not a set

The ordinary sheaf condition only gives locally constant maps $M \to BG$.

<u>Idea</u>: enlarge the diagram to

$$\operatorname{holim}\left(\prod_{i_1,i_2,i_3} F(U_{i_1} \cap U_{i_2} \cap U_{i_3}) \substack{\leftarrow\\\leftarrow\\\leftarrow} \prod_{j_1,j_2} F(U_{j_1} \cap U_{j_2}) \substack{\leftarrow\\\leftarrow} \prod_k F(U_k) \right)$$

(Remark for experts: sheaf $\Leftarrow$ hypercomplete sheaf in this case)

Def $F : \mathrm{Mfd}^{op} \to Sp$ is a <u>differential cohomology theory</u> if the following canonical map is an equivalence:

$$F(M) \xrightarrow{\sim} \operatorname{holim}_{\Delta}\left(\cdots \substack{\leftarrow\\\leftarrow\\\leftarrow} \prod_{i_1,i_2} F(U_{i_1} \cap U_{i_2}) \substack{\leftarrow\\\leftarrow} \prod_i F(U_i) \right)$$

(Čech nerve $U_\bullet : \Delta^{op} \to \mathrm{Mfd}$ simplicial manifold

$U_n = \coprod_{i_1,\dots,i_n} U_{i_1} \cap \dots \cap U_{i_n}$)

$\Rightarrow F(U_\bullet)$ cosimplicial spectrum

Given F, the <u>differential cohomology</u> of $M \in \mathrm{Mfd}$ is

$$\hat{H}^k(M, F) = \pi_{-k}(F(M))$$

Example If $E \in Sp$, define the <u>constant sheaf</u>

$$\mathrm{const}\, E\,(M) = \mathrm{Map}_{Sp}(\Sigma^\infty M_+, E) \in Sp$$

$$\Rightarrow \hat{H}^K(M, \mathrm{const}\, E) = H^K(M, E)$$

Prop $\exists$ Adjunction

$$Sh(Mfd, Sp) \underset{ev_{pt}}{\overset{\mathrm{const}}{\rightleftarrows}} Sp$$

Example For G compact Lie, $F = B^\nabla G$ does

$$M \mapsto \{\text{principal } G\text{-bundles}_{\text{on } M} \text{ with connection}\}$$

$$\text{Groupoids} = \text{homotopy 1-types} \subseteq \text{Spaces} \xrightarrow{\Sigma^\infty} Sp$$

Lemma If $A^\bullet : \Delta \to Ch$ is a cosimplicial chain complex,

then $\underset{\Delta}{\mathrm{holim}}\, DK(A^\bullet) = DK(\mathrm{Tot}\, A)$

where the <u>totalization</u> has

$$\mathrm{Tot}\, A[n] = \prod_{p-q=n} A^p[q]$$

Cor If $A : \mathrm{Mfd} \to \mathrm{Ch}$ is a complex of sheaves s.t. for all $k \in \mathbb{Z}$, $A[k]$ is a C^∞-module, then $DK \circ A$ is a differential cohomology theory

Example de Rham forms are a sheaf of chain complexes

$$\Omega^\bullet : \mathrm{Mfd} \to \mathrm{Ch} \xrightarrow{\text{Dold-Kan}} \mathrm{Sp}$$

Dold-Kan sends quasi-isomorphic chain complexes to equivalent spectra, so

$\text{Dold-Kan}(\Omega^\bullet) \simeq \mathrm{const}\, H\mathbb{R}$

By the de Rham theorem

4. Homotopy Invariance & Homotopification

Def A differential cohomology theory F is called <u>homotopy invariant</u> (also <u>$\mathbb{R}$-invariant</u> or <u>concordance invariant</u>) if the map

$$F(M) \to F(M \times \mathbb{R})$$

induced by the projection is an equivalence $\forall M \in \mathrm{Mfd}$.

$\Rightarrow$ suspension axiom & homotopy invariance

Th The adjunction

$$\begin{tikzcd} \mathrm{Sh}(\mathrm{Mfd}, \mathrm{Sp}) \arrow[r, bend right, "ev_{pt}"'] & \mathrm{Sp} \arrow[l, bend right, "\mathrm{const}"'] \end{tikzcd}$$

restricts to an equivalence on homotopy-invariant sheaves.

Example Recall that

$$\text{Dold-Kan}(\Omega^\bullet) \simeq \mathrm{const}\, H\mathbb{R}$$

which is homotopy invariant

$\Rightarrow$ need to do more than $\Omega^\bullet$ to get differential cohomology theories that contain geometric information.

Example for $k>0$, let $\Omega^{\geq k}$ be the "truncation bête"

$$\cdots \to 0 \to \Omega^{k}(M) \xrightarrow{d} \Omega^{k+1}(M) \to \cdots$$

It is a sheaf. This is <u>not</u> a Postnikov type truncation, but a "stupid truncation".

The differential cohomology of $M \in \mathrm{Mfd}$ with coefficients in $DK(\Omega^{\geq k})$ is

$$\hat{H}^n(M, DK(\Omega^{\geq k})) = \pi_{-n}(DK(\Omega^{\geq k}(M)))$$

$$= H^n(\Omega^{\geq k}(M)) = \begin{cases} 0 & n<k \\ \Omega^k_{cl}(M) & n=k \\ H^n(M,\mathbb{R}) & n>k \end{cases}$$

- $DK(\Omega^{\geq k})$ is not homotopy invariant.

$\Rightarrow$

- $DK(\Omega^{\geq k})$ contains geometric data similar to Deligne cohomology.

Ex Similarly, the truncation $\Omega^{\leq k}$ has

$$H^n(\Omega^{\leq k}(M)) = \begin{cases} H^n(M,\mathbb{R}) & n < k \\ \dfrac{\Omega^k(M)}{\operatorname{Im} d} & n = k \\ 0 & n > k \end{cases}$$

"differential deformations"

Def Deligne cohomology in dimension k is the pull back

$$\begin{tikzcd}
\mathrm{Del}_k \arrow[r, "u"] \arrow[d, "R"'] & \text{const } H\mathbb{Z} \arrow[d] \\
\Omega^{\geq k} \arrow[r] \arrow[dr, hook] & \text{const } H\mathbb{R} \arrow[d, phantom, "\simeq"] \\
& \Omega^{\bullet}
\end{tikzcd}$$

Def/Th There is a further left adjoint

$$\begin{tikzcd} \mathrm{Sh}(\mathrm{Mfd}, \mathrm{Sp}) \arrow[r, bend left, "\mathcal{H}"] \arrow[r, bend right, "\mathrm{ev}_{pt}"'] & \mathrm{Sp} \arrow[l, "\mathrm{const}"'] \end{tikzcd}$$

called <u>homotopification</u> (or <u>$\mathbb{R}$-localization</u> or <u>concordification</u>). If $F \in \mathrm{Sh}(\mathrm{Mfd}, \mathrm{Sp})$ is a differential cohomology theory, then

- $\mathcal{H}(F)$ is called the <u>underlying cohomology theory</u>

  We also say F is a <u>differential refinement</u> of $\mathcal{H}(F)$

- $F(pt)$ is the corresponding <u>secondary</u> or <u>flat</u> <u>cohomology theory</u>

Prop Concretely,

$$\mathcal{H}(F) = \operatorname{hocolim}_{\Delta^{op}} \overbrace{F(\underbrace{\Delta^\bullet}_{\text{cosimplicial manifold}})}^{\text{Simplicial spectrum}}$$

where $\Delta^n = \{ (t_0, \dots, t_n) \in \mathbb{R}^{n+1} : t_0 + \dots + t_n = 1 \}$

Example Consider $N \in \mathrm{Mfd}$ as a diffeological space

$$F = C^\infty(-,N) : \mathrm{Mfd}^{op} \to \mathrm{Set} \subseteq \mathrm{Spaces} \xrightarrow{\Sigma^\infty} \mathrm{Sp}.$$

The corresponding flat theory is

$$F(pt) = \Sigma^\infty N_+^\delta \qquad BG^\delta$$

The underlying cohomology theory is

$$\mathcal{H}F = \Sigma^\infty N_+ \qquad BG$$

Example For G cpt Lie group

$$F(M) = \{ G\text{-principal bundles over } M \}$$
$$\in \mathrm{Groupoids} \subseteq \mathrm{Spaces}_* \xrightarrow{\Sigma^\infty} \mathrm{Sp}$$

Then

Flat theory: $F(pt) = \Sigma^\infty BG^\delta$

Underlying theory: $\mathcal{H}F = \Sigma^\infty BG$

Similar for $F = B^\nabla G$

Lemma If $A^\bullet : \Delta^{op} \to Ch$ is a simplicial chain complex, then $\operatorname{holim}_{\Delta^{op}} DK(A^\bullet) = DK(\mathrm{Tot}\, A)$

where the totalization has

$$\mathrm{Tot}\, A[n] = \prod_{p+q=n} A^p[q]$$

Cor Let $A \in Sh(Mfd, Ab)$ be a sheaf of C^∞-modules. Then $\mathcal{H}(DK(A[0])) \simeq 0$

Cor Let $A \in sh(Mfd, Ch)$ sheaf of chain complexes s.t. $A[k]$ is a C^∞-module for every k. Then

1. $\mathcal{H}(DK(A^{\geq k})) \simeq \mathcal{H}(DK(A))$
2. $\mathcal{H}(DK(A^{\leq k})) \simeq 0$

Th Deligne cohomology has

$$\mathcal{H}(Del_k) = H\mathbb{Z}$$

So Deligne cohomology is a differential refinement of integral cohomology

Note $\mathrm{Del}_k(pt) = \Sigma^{-1} H(\mathbb{R}/\mathbb{Z})$ if $k>0$

since

$$\begin{tikzcd}
\mathrm{Del}_k(pt) \arrow[r, "u"] \arrow[d, "R"'] & H\mathbb{Z} \arrow[d] \\
\Omega^{\geq k}(pt) \arrow[r] & H\mathbb{R}
\end{tikzcd}$$

$\Omega^{\geq k}(pt) = 0$

Bonus Invertible Smooth Field Theories

Let N be a smooth set such as a manifold or $B^{\nabla}G$.

Given a class $\alpha \in \hat{H}^{n+1}(N)$ one can define a n-dimensional "sigma model with target N" by the "action functional"

$$S(\phi : M^n \to N) = \int_M \phi^* \alpha \in \mathbb{R}/\mathbb{Z}$$

oriented

where $\phi^*\alpha \in \hat{H}^{n+1}(M) \cong H^n(M, \mathbb{R}/\mathbb{Z})$

M n-dim

<u>Th</u> (Freed-Hopkins)

Chern-Weyl

$$\Omega^{2n}(B^{\nabla}G) = \left\{ \begin{matrix} G\text{-invariant polynomials} \\ \text{on } \mathfrak{g} \text{ of degree } n \end{matrix} \right\} \cong H^{2n}(BG, \mathbb{R})$$

and $\Omega^{\bullet}(B^{\nabla}G)$ has trivial differential

$\Rightarrow \hat{H}^{2n}(B^{\nabla}G)$ consists of G-invariant polynomials together with a chosen integer lift of its Chern Weyl representative

$\Rightarrow$ defines the Chern-Simons action in $(2n-1)$-dimensions.

Outlook: Fracture Square & Recollement

$$
\begin{tikzcd}
 & \mathrm{Const}\, F(pt) \arrow[r, phantom, "\simeq"] \arrow[d] & \mathrm{const}\, F(pt) \arrow[d] & \\
\mathcal{A}(F) \arrow[r, "a"] \arrow[d, equal] & F \arrow[r, "u"] \arrow[d, "R"'] \arrow[dr, phantom, "\lrcorner", very near start] & \mathrm{const}\, \mathcal{H} F \arrow[d] & \sim H\mathbb{Z} \\
\mathcal{A}(F) \arrow[r, "d"] \arrow[d, phantom, "\simeq" sloped] & Z(F) \arrow[r] \arrow[d, phantom, "\simeq" sloped] & \mathrm{Const}(\mathcal{H} Z(F)(pt)) \arrow[d, phantom, "\simeq" sloped] & \\
\Omega^{k-1}/\mathrm{Im}\, d & \Omega^{k}_{cl} & H\mathbb{R} &
\end{tikzcd}
$$

Def F is <u>purely geometric</u> if $F(pt) \sim *$

<u>Idea</u> Purely geometric sheaves are an 'orthogonal complement' of homotopy-invariant sheaves:

F is purely geometric $\Leftrightarrow$ $\forall$ homotopy-invariant F'

$$\mathrm{Map}_{Sh(Man, Sp)}(F', F) \simeq *$$

Def The <u>underlying cohomology class natural transformation</u>

$$u : \mathrm{id}_{Sh(Mfd, Sp)} \to \mathrm{const} \circ \mathcal{H}$$

is the unit of the adjunction

$$\begin{tikzcd} Sh(Mfd, Sp) \arrow[r, bend left, "\mathcal{H}"] & Sp \arrow[l, bend left, "\mathrm{const}"] \end{tikzcd}$$

Def The <u>differential deformations</u> $A(F) \in Sh(Mfd, Sp)$ of a differential cohomology theory F are defined by applying the functor $A : Sh(Mfd, Sp) \to Sh(Mfd, Sp)$ which is the fiber

$$A \to \mathrm{id}_{Sh(Mfd, Sp)} \xrightarrow{u} \mathrm{const}\, \mathcal{H}$$

More Extra Stuff

Note: Ω^k_{cl} is not a C^∞-module

in fact $DK(\Omega^k_{cl}[k]) : MFd \to Sp$ is not a sheaf.

Its sheafification is $DK(\Omega^{\geq k})$, because locally the inclusion

$$\begin{tikzcd}
\cdots \arrow[r] & 0 \arrow[r] \arrow[d] & \Omega^k_{cl}(U) \arrow[r] \arrow[d] & 0 \arrow[r] \arrow[d] & \cdots \\
\cdots \arrow[r] & 0 \arrow[r] & \Omega^k(U) \arrow[r] & \Omega^{k+1}(U) \arrow[r] & \cdots
\end{tikzcd}$$

is a quasi-iso

Remark There is also a further right adjoint

$$\begin{tikzcd}
Sh(MFd, Sp) \arrow[r, bend left, "\mathcal{H}"] \arrow[r, bend right, "ev_{pt}"'] & Sp \arrow[l, "const"'] \arrow[l, bend left=80, "disc"]
\end{tikzcd}
\qquad (disc\, X)(M) = Map_{Sp}(\Sigma^\infty M^\delta, X)$$

called the <u>Godement functor</u>. This is important for <u>'cohesion'</u>.