Faculty of Science Syllabus

Department of Mathematics and Statistics

Math 5160, Operator Algebras, Fall 2025

Dalhousie University acknowledges that we are in Mi'kma'ki, the ancestral and unceded territory of the Mi'kmaq People and pays respect to the Indigenous knowledges held by the Mi'kmaq People, and to the wisdom of their Elders past and present. The Mi'kmaq People signed Peace and Friendship Treaties with the Crown, and section 35 of the Constitution Act, 1982 recognizes and affirms Aboriginal and Treaty rights. We are all Treaty people.

Dalhousie University also acknowledges the histories, contributions, and legacies of African Nova Scotians, who have been here for over 400 years.

Course Instructor

Name	Email	Office Hours
Luuk Stehouwer	luuk@dal.ca	Wednesdays 14:35-15:25, Chase Building, room 224

Course Description

Many familiar mathematical objects come equipped with algebraic as well as topological structure. For example, there is a space of matrices because one can define the 'distance' between two matrices using the operator norm. In this course we will explore the wonderful algebraic and functional analytic properties of such algebras of operators.

In particular, we will explore how commutative operator algebras can be seen as functions on certain topological spaces and measure spaces. An essential tool is the spectrum of an operator, a generalization of the notion of eigenvalue. We will take a special look at von Neumann algebras, which are classified by their decomposition into mysterious irreducible components called factors.

Course Prerequisites

Permission of the instructor

Students are assumed to be familiar with basic ring theory (such as ring homomorphisms, matrix rings and ideals), as well as basic analysis (such as limits of sequences, continuity and inner product spaces). Basic understanding of point-set topological concepts such as metric spaces, function spaces, compactness and completeness is recommended.

Course Structure

Course Delivery

Lectures will be given in person. Attendance is not mandatory. None of the lectures will be recorded.

Lectures

Lectures take place Tuesdays and Thursdays 11:35-12:55 in Chase 227.

Tutorials

There will be no tutorials

Course Materials

For the first part of the course on C*-algebras, we will use Chapter 2 of the publicly available PhD thesis "The Category of Von Neumann Algebras" by Abraham A. Westerbaan. In the second part about von Neumann algebras we will use a variety of sources:

1. Chapter 3 of the aforementioned PhD thesis
2. David Penneys' notes on von Neumann algebras and factors available at https://people.math.osu.edu/penneys.2/7211/2024/Math7212.html
3. Groundwork For Operator Algebras Lecture Series' notes on von Neumann algebras available at https://users.math.msu.edu/users/banelson/conferences/GOALS/curriculum.html
4. Jacob Lurie's notes on von Neumann algebras

Assessment

Assignments

There will be exercises to hand in - roughly every other week - contributing to 30% of the final grade.

It is recommended to work together on the exercises, but every student must write their homework individually. I recommend looking for online sources to solve your problem, but directly asking for help online is not allowed.

Note that generative AI might be able to convince you it understands your problem, but in practice it can at best copy the knowledge available on the internet. Understanding is reached by patiently struggling with pen and paper.

The assignment schedule is as follows:

- **Assignment 1** (Due: September 18) - Hilbert spaces and C*-algebras
- **Assignment 2** (Due: September 25) - Spectral theory and positive elements
- **Assignment 3** (Due: October 9) - Gelfand representation
- **Assignment 4** (Due: October 23) - von Neumann algebras
- **Assignment 5** (Due: October 30) - Constructions of factors
- **Assignment 6** (Due: November 20) - Measure theory and commutative von Neumann algebras

Midterm and final exam

There will be a midterm worth 20% the week of October 13. There will be a final exam during the exam period at the end of the semester for 35%.

Other course requirement: presentation

Each student will give a short presentation (10-20 minutes) on an advanced topic related to von Neumann algebras. The presentation will contribute 15% of the final grade. A list of possible topics will be provided, but students are encouraged to choose a topic of interest with approval from the instructor. The presentations will be planned at the end of October and take place throughout November. The presentation will be graded based on the following criteria:

- Mathematical Content (40%): The presentation demonstrates a solid understanding of the chosen topic. This involves giving correct definitions, theorems, and proofs.
- Clarity and Structure (30%): The presenter explains key results in an understandable way. The presentation is well-organized, with a logical

sequence of ideas. The speaker provides context through an introduction, and the main body develops the topic systematically.

- Engagement and Delivery (15%): The speaker communicates effectively, shows skilled blackboard usage and keeps the audience engaged.
- Response to Questions (15%): The presenter should demonstrate the ability to respond to audience questions with understanding and clarity. It will be more important to respond thoughtfully rather than have the ability to provide a full answer.

Conversion of numerical grades to final letter grades follows the Dalhousie Graduate Grade Scale

A+ (90-100)	B+ (77-79)	
A (85-89)	B (73-76)	F (0-69)
A- (80-84)	B- (70-72)	

Course Policies on Missed or Late Academic Requirements

Late assignments will incur a penalty of 10% per day, up to a maximum of three days. After this period, assignments will not be accepted unless prior arrangements are made with the instructor. If a student misses an assignment deadline or their presentation due to illness or other valid reasons, they must submit a Student Declaration of Absence form. Each student may use this form up to two times during the term.

Missed final exams will be handled in accordance with university policy. Students who miss the final exam due to valid reasons must provide documentation and may be allowed to take a deferred exam at the discretion of the instructor.

Learning Objectives

By the end of the course, students will:

- Understand the fundamentals of C*-algebras and their functional analytic properties.
- Develop a working knowledge of the Gelfand representation, the spectrum and functional calculus in order to apply them in examples.
- Construct and write simple proofs involving operator algebras.
- Understand the role of projections and the structure of factors in the classification of von Neumann algebras.

- Obtain sufficient prerequisites to be able to read about advanced topics such as subfactors, bimodules, and Connes fusion.
- Present mathematical ideas clearly and effectively through an oral presentation.

Course Content

The first half of the course will be an introduction to C*-algebras. After constructing the functional calculus, we discuss how to recover the norm of an element from its spectrum. Our main goal will be to prove the Gelfand representation theorem saying that every commutative C*-algebra is the algebra of functions on a compact Hausdorff space. As time permits, we will discuss the Gelfand-Naimark-Segal construction and the Gelfand-Naimark theorem.

In the second part of the course we will specialize to von Neumann algebras. We will show that elements of von Neumann algebras have a polar decomposition. Commutative von Neumann algebras are given by functions on a measure space. In the other extreme, von Neumann algebras with trivial centers are called factors, which are classified into several types. We will see that general von Neumann algebras are essentially a combination of these two types. We will construct the predual of a von Neumann algebra and study normal functionals. An essential ingredient in this theory is the behavior of projectors in a von Neumann algebra. As time permits, we will study subfactors, Lp-spaces, bimodules and Connes fusion.

University Policies and Statements

For University Policies, Guidelines and Resources for Learning, we refer to Section C of the Course Syllabus Guide: https://cdn.dal.ca/content/dam/dalhousie/pdf/dept/clt/SECTION%20C%20Syllabus%20Policy.pdf

Recognition of Mi'kmaq Territory

Dalhousie University would like to acknowledge that the University is on Traditional Mi'kmaq Territory. The Elders in Residence program provides students with access to First Nations elders for guidance, counsel, and support. Visit or e-mail the Indigenous Student Centre at 1321 Edward St or elders@dal.ca. Additional information regarding the Indigenous Student Centre can be found at: https://www.dal.ca/campus_life/communities/indigenous.html

Internationalization

At Dalhousie, 'thinking and acting globally' enhances the quality and impact of education, supporting learning that is "interdisciplinary, cross-cultural, global in reach, and orientated toward solving problems that extend across national borders." Additional internationalization information can be found at: https://www.dal.ca/about-dal/internationalization.html

Academic Integrity

At Dalhousie University, we are guided in all our work by the values of academic integrity: honesty, trust, fairness, responsibility, and respect. As a student, you are required to demonstrate these values in all the work you do. The University provides policies and procedures that every member of the university community is required to follow to ensure academic integrity. Additional academic integrity information can be found at: https://www.dal.ca/dept/university_secretariat/academic-integrity.html

Accessibility

The Student Accessibility Centre is Dalhousie's centre of expertise for matters related to student accessibility and accommodation. If there are aspects of the design, instruction, and/or experiences within this course (online or in-person) that result in barriers to your inclusion, please contact the Student Accessibility Centre (https://www.dal.ca/campus_life/academic-support/accessibility.html) for all courses offered by Dalhousie with the exception of Truro. For courses offered by the Faculty of Agriculture, please contact the Student Success Centre in Truro (https://www.dal.ca/about-dal/agricultural-campus/student-success-centre.html)

Conduct in the Classroom – Culture of Respect

Substantial and constructive dialogue on challenging issues is an important part of academic inquiry and exchange. It requires willingness to listen and tolerance of opposing points of view. Consideration of individual differences and alternative viewpoints is required of all class members, towards each other, towards instructors, and towards guest speakers. While expressions of differing perspectives are welcome and encouraged, the words and language used should remain within acceptable bounds of civility and respect.

Diversity and Inclusion – Culture of Respect

Every person at Dalhousie has a right to be respected and safe. We believe inclusiveness is fundamental to education. We stand for equality. Dalhousie is strengthened in our diversity. We are a respectful and inclusive community. We are committed to being a place where everyone feels welcome and supported, which is why our Strategic Direction prioritizes fostering a culture of diversity and inclusiveness (Strategic Priority 5.2). Additional diversity and inclusion information can be found at: http://www.dal.ca/cultureofrespect.html

Student Code of Conduct

Everyone at Dalhousie is expected to treat others with dignity and respect. The Code of Student Conduct allows Dalhousie to take disciplinary action if students don't follow this community expectation. When appropriate, violations of the code can be resolved in a reasonable and informal manner - perhaps through a restorative justice process. If an informal resolution can't be reached, or would be inappropriate, procedures exist for formal dispute resolution. The full Code of Student Conduct can be found at: https://www.dal.ca/dept/university_secretariat/policies/student-life/code-of-student-conduct.html

Fair Dealing Policy

The Dalhousie University Fair Dealing Policy provides guidance for the limited use of copyright protected material without the risk of infringement and without having to seek the permission of copyright owners. It is intended to provide a balance between the rights of creators and the rights of users at Dalhousie. Additional information regarding the Fair Dealing Policy can be found at: https://www.dal.ca/dept/university_secretariat/policies/academic/fair-dealing-policy-.html

Originality Checking Software

The course instructor may use Dalhousie's approved originality checking software and Google to check the originality of any work submitted for credit, in accordance with the Student Submission of Assignments and Use of Originality Checking Software Policy. Students are free, without penalty of grade, to choose an alternative method of attesting to the authenticity of their work and must inform the instructor no later than the last day to add/drop classes of their intent to choose an alternate method. Additional information regarding Originality Checking Software can be found at: https://www.dal.ca/about/leadership-governance/academic-integrity/faculty-resources/ouriginal-plagiarism-detection.html

Student Use of Course Materials

Course materials are designed for use as part of this course at Dalhousie University and are the property of the instructor unless otherwise stated. Third party copyrighted materials (such as books, journal articles, music, videos, etc.) have either been licensed for use in this course or fall under an exception or limitation in Canadian Copyright law. Copying this course material for distribution (e.g. uploading to a commercial third-party website) may lead to a violation of Copyright law.